Section 4.5

# Solving Systems Using Inverse Matrices

# Writing a Matrix Equation

Solve the system of linear equations

$$5x + 2y = 3$$
$$4x + 2y = 4$$

$$\begin{array}{ccc} A & X & B \\ \begin{bmatrix} 5 & 2 \\ 4 & 2 \end{bmatrix} & \begin{bmatrix} x \\ y \end{bmatrix} = & \begin{bmatrix} 3 \\ 4 \end{bmatrix} \end{array}$$

Write the system as a matrix equation

- Matrix A is the coefficient matrix of the system
- X is the **matrix of variables**
- B is the **matrix of constants**

**Matrix Equation:** $AX = B$

# Solving the Matrix Equation

1) Write the original equation
2) Multiply each side by $A^{-1}$
3) $A^{-1} \cdot A = I$
4) $IX = X$

1) $AX = B$
2) $A^{-1} \cdot AX = A^{-1} \cdot B$
3) $IX = A^{-1} \cdot B$
4) $X = A^{-1} \cdot B$

# Solving a Linear System

Use matrices to solve the linear system

$|A| = 2$

$$5x + 2y = 3$$
$$4x + 2y = 4$$

$$A^{-1} = \frac{1}{2}\begin{bmatrix} 2 & -2 \\ -4 & 5 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ -2 & 5/2 \end{bmatrix}$$

**Steps to follow**

1. Write the system in matrix form
2. Find the inverse of matrix A
3. Multiply the $\underbrace{\text{matrix of constants}}_{B}$ by $A^{-1}$
4. Check the solution in the original equations

$$\begin{bmatrix} 5 & 2 \\ 4 & 2 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 \\ 4 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} 1 & -1 \\ -2 & \frac{5}{2} \end{bmatrix}$$

$$A^{-1} \cdot B = \begin{bmatrix} 1 & -1 \\ -2 & \frac{5}{2} \end{bmatrix} \begin{bmatrix} 3 \\ 4 \end{bmatrix}$$

$$= \begin{bmatrix} -1 \\ 4 \end{bmatrix}$$

The solution is $(-1, 4)$

# Use an Inverse Matrix to Solve the Linear System

$$2x - y = 1$$
$$-3x + 2y = 0$$

$$\underbrace{\begin{bmatrix} 2 & -1 \\ -3 & 2 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} x \\ y \end{bmatrix}}_{X} = \underbrace{\begin{bmatrix} 1 \\ 0 \end{bmatrix}}_{B}$$

$X = B$

$|A| = 4 - 3 = 1$

$A^{-1} = \frac{1}{1}\begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix}$

$\begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 2\cdot 1 + 1\cdot 0 \\ 3\cdot 1 + 2\cdot 0 \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$

$(2, 3)$

$$2x + 6y - 4z = -10$$
$$-3x + 10y - 7z = -23$$
$$-2x - 6y + 5z = 10$$

$$\underbrace{\begin{bmatrix} 2 & 6 & -4 \\ -3 & 10 & -7 \\ -2 & -6 & 5 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} x \\ y \\ z \end{bmatrix}}_{X} = \underbrace{\begin{bmatrix} -10 \\ -23 \\ 10 \end{bmatrix}}_{B}$$

$(1, -2, 0)$

# Application

You can purchase peanuts for $3 per pound, almonds for $4 per pound, and cashews for $8 per pound. You want to create 140 pounds of a mixture that costs $6 per pound. If twice as many peanuts are used than almonds, how many pounds of each type should be used?

$3p + 4a + 8c =$

# Application

$\cancel{2y = z}$ $2z = y$

A chemist wants to use three different solutions to create a 50-liter mixture containing 32% acid. The first solution contains 10% acid, the second 30%, and the third 50%. He needs to use twice as much of the 50% solution as the 30% solution. How many liters of each solution should be used?

$.32(50)$

$$.1x + .3y + .5z = 16$$

$$x + y + z = 50$$

$$-y + 2z = 0$$

$$\underbrace{\begin{bmatrix} .1 & .3 & .5 \\ 1 & 1 & 1 \\ 0 & -1 & 2 \end{bmatrix}}_{A} \cdot \underbrace{\begin{bmatrix} x \\ y \\ z \end{bmatrix}}_{X} = \underbrace{\begin{bmatrix} 16 \\ 50 \\ 0 \end{bmatrix}}_{B}$$

$$A^{-1} \cdot B = X$$

$$\begin{bmatrix} 8.75 \\ 27.5 \\ 13.75 \end{bmatrix}$$

# Homework

- Textbook page 233 #11, 23, 27 (show work by hand)
- Textbook page 234 #34, 39 (on calculator)
- Textbook page 235 #46 (on calculator)